



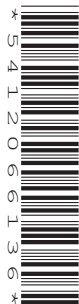
Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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1 Find the set of values of k for which the system of equations

$$x + 5y + 6z = 1,$$

$$kx + 2y + 2z = 2,$$

$$-3x + 4y + 8z = 3,$$

has a unique solution and interpret this situation geometrically. [4]

[4]

[illegible]



2 It is given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = \cos^{-1} t \quad \text{for } 0 < t < 1.$$

(a) Show that $\frac{dy}{dx} = \frac{t^2}{\sqrt{1-t^2}}$. [2]

[illegible]



(b) Show that $\frac{d^2y}{dx^2} = -t^a(1-t^2)^b(2-t^2)$, where a and b are constants to be determined. [4]

[illegible]



- 3 A curve has equation $y = e^x$ for $\ln \frac{4}{3} \leq x \leq \ln \frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by A .

(a) Use the substitution $u = e^x$ to show that

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2} \, du. \quad [2]$$

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(b) Use the substitution $u = \sinh v$ to show that

$$A = \pi \left(\frac{904}{225} + \ln \frac{5}{3} \right). \quad [6]$$

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(c) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$.

[4]

[illegible]



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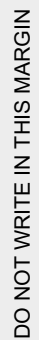
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- (b) Use a similar method to find, in terms of N , an upper bound U_N for $\int_0^1 \left(\frac{1}{2}\right)^x dx$. [4]

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- (c) Find the least value of N such that $U_N - L_N \leq 10^{-3}$. [2]

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- (d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x dx = \frac{1}{2\ln 2}$, use the value of N found in part (c) to find upper and lower bounds for $\ln 2$. [4]

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- 7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

is $\frac{1}{4}x + \frac{1}{4}\sqrt{x^2 + 16}$. [4]

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(b) Hence find the solution of the differential equation

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

for which $y = 6$ when $x = 3$.

[6]

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- 8 (a)** By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^7$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cos^7 \theta = a \cos 7\theta + b \cos 5\theta + c \cos 3\theta + d \cos \theta,$$

where a, b, c and d are constants to be determined. [5]

[illegible]





$$nI_n = 2^{-\frac{1}{2}n} + (n-1)I_{n-2}. \quad [4]$$
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(c) Using the results given in parts (a) and (b), find the exact value of I_9 .

[5]

[illegible]



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